Note Title 1/25/2008

$\psi(x)$ has either even or odd parity: $\psi(x) = \pm\psi(-x) \Rightarrow$

Even parity:

$$\psi(x) = \begin{cases} Ce^{Kx} & x < -L \\ B\cos kx & -L < x < L \\ Ce^{-Kx} & x > L \end{cases}$$

Odd parity:

$$\psi(x) = \begin{cases} Ce^{Kx} & x < -L \\ A\sin kx & -L < x < L \\ -Ce^{-Kx} & x > L \end{cases}$$

Apply BC's: ψ & ψ' are continous

Since we already applied the symmetry through parity, we only need to apply the B.C.'s on one side, say at $x = L$:

For ψ with Even parity we have:

$$\psi(L^-) = \psi(L^+) \rightarrow \quad B\cos kL = Ce^{-KL}$$

$$\psi'(L^-) = \psi'(L^+) \rightarrow \quad -Bk\sin kL = -CKe^{-KL}$$

$\rightarrow k \tan kL = K$ or: $\boxed{kL \tan kL = KL}$

Even parity

For odd parity ψ:

$\psi(L^-) = \psi(L^+) \rightarrow A \sin kL = -Ce^{-KL}$

$\psi'(L^-) = \psi(L^+) \rightarrow Ak \cos kL = CK e^{-KL}$

$\rightarrow k \cot kL = -K$ or: $\boxed{kL \cot kL = -KL}$

odd parity

Recall: $E = \frac{\hbar^2 k^2}{2m}$, $V_0 - E = \frac{\hbar^2 K^2}{2m}$ $\Big\} \; E + V_0 - E = \frac{\hbar^2}{2m}(k^2 + K^2)$

Or: $(kL)^2 + (KL)^2 = \underbrace{\frac{2mV_0}{\hbar^2}}_{K_0^2} L^2$ i.e. a constant

$$(kL)^2 + (KL)^2 = (K_0 L)^2$$